

## Reciprocity and orthogonality

Chandan Singh Dalawat  
Harish-Chandra Research Institute  
Chhatnag Road, Jhansi, Allahabad 211019, India  
dalawat@gmail.com

**Abstract.** Let  $p$  be a prime and let  $K$  be a finite extension of the field  $\mathbf{Q}_p$  of  $p$ -adic numbers such that the group  ${}_pK^\times$  has order  $p$ . The  $\mathbf{F}_p$ -space  $K^\times/K^{\times p}$  carries a natural filtration coming from the valuation on  $K$ , and a natural bilinear pairing coming from the reciprocity isomorphism for the exponent  $p$ . We determine the orthogonal filtration for this pairing. We also prove the analogous result for  $p$ -fields of characteristic  $p$ .

(1) Let  $p$  be a prime number and let  $K$  be a  $p$ -field — a field complete for a discrete valuation with finite residue field of characteristic  $p$ . Let  $M$  be the maximal abelian extension of  $K$  of exponent  $p$ ,  $G = \text{Gal}(M|K)$ ,  $\overline{K^\times} = K^\times/K^{\times p}$ , and  $\rho : \overline{K^\times} \rightarrow G$  the reciprocity isomorphism (for the exponent  $p$ ). If  $K$  has characteristic 0 and  ${}_pK^\times$  has order  $p$ , then there is a natural pairing

$$G \times \overline{K^\times} \rightarrow {}_pK^\times, \quad (\sigma, \bar{x}) \mapsto \sigma(y)y^{-1} \quad (y \in M, y^p = x).$$

Similarly, if  $K$  has characteristic  $p$ , then there is a natural pairing

$$G \times \overline{K^+} \rightarrow \mathbf{F}_p, \quad (\sigma, \bar{x}) \mapsto \sigma(y) - y \quad (y \in M, \wp(y) = x),$$

where  $\overline{K^+} = K^+/\wp(K^+)$  and  $\wp(z) = z^p - z$  for  $z$  in any  $\mathbf{F}_p$ -algebra.

(2) When combined with the reciprocity isomorphism  $\rho : \overline{K^\times} \rightarrow G$ , we get the *hilbertian pairing*

$$\overline{K^\times} \times \overline{K^\times} \rightarrow {}_pK^\times \quad (\text{resp. } \overline{K^\times} \times \overline{K^+} \rightarrow \mathbf{F}_p).$$

The main aim of this Note is to show that the filtrations on the two factors, coming from the filtration(s) on  $K^\times$  (resp. on  $K^\times$  and  $K^+$ ), are orthogonal to each other for this pairing. This will be soon made precise.

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## 1. Notation

(3)  $(p, K, \mathfrak{o}, \mathfrak{p}, k)$ . Throughout,  $p$  is a prime number and  $K$  is a  $p$ -field — a field complete for a discrete valuation with finite residue field of characteristic  $p$ . The ring of integers of  $K$  is  $\mathfrak{o}$ , the unique maximal ideal of  $\mathfrak{o}$  is  $\mathfrak{p}$ , and the residue field is  $k = \mathfrak{o}/\mathfrak{p}$ .

(4)  $(e, f)$ . The (absolute) residual degree of  $K$  is  $f = [k : \mathbf{F}_p]$ . If  $K$  is an extension of  $\mathbf{Q}_p$ , then  $e = [K : \mathbf{Q}_p]f^{-1}$  is the (absolute) ramification index ; if  $K$  has characteristic  $p$ , then  $e = +\infty$ . We often say “ $e < +\infty$ ” to mean that  $K$  has characteristic 0.

(5)  $(\overline{K^\times}, \bar{U}_i)$ . For every  $i > 0$ , let  $U_i = 1 + \mathfrak{p}^i$  be the kernel of  $\mathfrak{o}^\times \rightarrow (\mathfrak{o}/\mathfrak{p}^i)^\times$  and denote the image of  $U_i$  in the  $\mathbf{F}_p$ -space  $\overline{K^\times} = K^\times/K^{\times p}$  by  $\bar{U}_i$ . Note that the image  $\mathfrak{o}^\times/\mathfrak{o}^{\times p}$  of  $\mathfrak{o}^\times$  in  $\overline{K^\times}$  is equal to  $\bar{U}_1$  because  $\mathfrak{o}^\times = U_1 \cdot k^\times$  and  $k^{\times p} = k^\times$ . The image of  $x \in K^\times$  in  $\overline{K^\times}$  is denoted  $\bar{x}$ .

(6)  $(\wp, \overline{K^+}, \bar{\mathfrak{p}}^i, \bar{\mathfrak{o}})$ . For any  $\mathbf{F}_p$ -algebra  $A$ , denote by  $\wp : A \rightarrow A$  the endomorphism  $\wp(x) = x^p - x$  of the additive group  $A^+$ . If  $e = +\infty$  (so that  $K$  is an  $\mathbf{F}_p$ -algebra), we put  $\overline{K^+} = K^+/\wp(K^+)$  and, for every  $i \in \mathbf{Z}$ , denote the image of  $\mathfrak{p}^i$  in the  $\mathbf{F}_p$ -space  $\overline{K^+}$  by  $\bar{\mathfrak{p}}^i$ . Sometimes we also denote  $\bar{\mathfrak{p}}^0$  by  $\bar{\mathfrak{o}}$  ; it equals  $\mathfrak{o}^+/\wp(\mathfrak{o}^+)$ . The image of  $x \in K^+$  in  $\overline{K^+}$  is denoted  $\bar{x}$ .

(7)  $(b_p^{(i)})$ . We denote by  $b_p^{(i)}$  ( $i > 0$ ) be the sequence of positive integers  $\not\equiv 0 \pmod{p}$ , namely  $b_p^{(i)} = i + \lfloor (i-1)/(p-1) \rfloor$ , where, for every  $x \in \mathbf{R}$ ,  $\lfloor x \rfloor$  is the largest integer in the interval  $]-\infty, x]$ .

(8)  $(c)$ . If  $e < +\infty$  and  $e \equiv 0 \pmod{p-1}$  (for example when  ${}_pK^\times$  has order  $p$ ), then we abbreviate  $c = (p-1)^{-1}e$ . Note that in this case  $pc = b_p^{(e)} + 1 = e + c$ .

(9)  $(v, \bar{v})$ . The surjective valuation  $K^\times \rightarrow \mathbf{Z}$  is denoted by  $v$  (so that  $e = v(p)$ ) ; it induces an isomorphism  $\bar{v} : \overline{K^\times}/\bar{U}_1 \rightarrow \mathbf{Z}/p\mathbf{Z}$ . For this reason, in order to determine the structure of the filtered  $\mathbf{F}_p$ -space  $\overline{K^\times}$ , it is enough to study the filtered  $\mathbf{F}_p$ -space  $\bar{U}_1$ .

## 2. The filtered $\mathbf{F}_p$ -space $\bar{U}_1$

(10) Let us determine the filtration on the  $\mathbf{F}_p$ -space  $\bar{U}_1$ . We will see that the dimension  $d = \dim_{\mathbf{F}_p} \bar{U}_1$  is finite or infinite according as  $e < +\infty$  or  $e = +\infty$ . When  $e < +\infty$ , one has  $d = [K : \mathbf{Q}_p] + \dim_{\mathbf{F}_p} ({}_pK^\times)$ , so there are two subcases according as  ${}_pK^\times$  is trivial or has order  $p$ . More precisely, by studying what the endomorphism  $(\ )^p : U_1 \rightarrow U_1$  does to the filtration on  $U_1$ , one determines the filtration on  $\bar{U}_1$  as follows.

(11) Suppose that  $e = +\infty$ . For  $i > 0$ , the inclusion  $\bar{U}_{i+1} \subset \bar{U}_i$  is an equality if  $i \equiv 0 \pmod{p}$ , and has codimension  $f$  if  $i \not\equiv 0 \pmod{p}$ .  $\square$

(12) Suppose that  $e < +\infty$  and  ${}_pK^\times$  is trivial. Then  $\bar{U}_i$  is trivial for  $i > b_p^{(e)}$ . For  $i \in [1, b_p^{(e)}]$ , the inclusion  $\bar{U}_{i+1} \subset \bar{U}_i$  is an equality if  $i \equiv 0 \pmod{p}$ , and has codimension  $f$  if  $i \not\equiv 0 \pmod{p}$ .  $\square$

(13) Suppose that  $e < +\infty$  and  ${}_pK^\times$  has order  $p$ . Then  $\bar{U}_i$  is trivial for  $i > pc$  and  $\bar{U}_{pc}$  has order  $p$ . For  $i \in [1, pc[$ , the inclusion  $\bar{U}_{i+1} \subset \bar{U}_i$  is an equality if  $i \equiv 0 \pmod{p}$ , and has codimension  $f$  if  $i \not\equiv 0 \pmod{p}$ .  $\square$

(14) *Remark.* Whenever  $\bar{U}_{i+1}$  has codimension  $f$  in  $\bar{U}_i$ , the quotient  $\bar{U}_i/\bar{U}_{i+1}$  is canonically isomorphic to  $U_i/U_{i+1}$ .

(15) Denote the reduction map  $\mathfrak{o} \rightarrow k$  by  $a \mapsto \hat{a}$  and let  $S : k \rightarrow \mathbf{F}_p$  be the trace map. If  $e < +\infty$  and if  $\zeta \in {}_pK^\times$  has order  $p$ , then  $v(p\pi) = pc$ , where  $\pi = 1 - \zeta$ , and the map  $\overline{1 + ap\pi} \mapsto \zeta^{S(\hat{a})}$  ( $a \in \mathfrak{o}$ ) is an isomorphism  $\bar{U}_{pc} \rightarrow {}_pK^\times$ , independent of  $\zeta$ . For proofs and more information, see [2, Prop. 42], for example.

### 3. The filtered $\mathbf{F}_p$ -space $\overline{K}^+$

(16) Suppose that  $e = +\infty$ . In analogy with the foregoing, by studying what the endomorphism  $\wp : K^+ \rightarrow K^+$  does to the filtration on  $K^+$  (by the powers  $\mathfrak{p}^i$  ( $i \in \mathbf{Z}$ ) of  $\mathfrak{p}$ ), one determines the filtration on  $\overline{K}^+$ .

(17) For every  $i > 0$ , one has  $\overline{\mathfrak{p}^i} = \{0\}$ . The group  $\overline{\mathfrak{p}^0} = \bar{\mathfrak{o}}$  has order  $p$ . For every  $i < 0$ , the inclusion  $\overline{\mathfrak{p}^{i+1}} \subset \overline{\mathfrak{p}^i}$  is an equality if  $i \equiv 0 \pmod{p}$ , and has codimension  $f$  if  $i \not\equiv 0 \pmod{p}$ .  $\square$

(18) *Remark.* Denote the passage to the quotient  $\mathfrak{o} \rightarrow \bar{\mathfrak{o}}$  (resp. the reduction map  $\mathfrak{o} \rightarrow k$ ) by  $x \mapsto \bar{x}$  (resp.  $x \mapsto \hat{x}$ ), and the trace map  $k \rightarrow \mathbf{F}_p$  by  $S$ . Then the map  $\bar{a} \mapsto S(\hat{a})$  ( $a \in \mathfrak{o}$ ) is an isomorphism  $\bar{\mathfrak{o}} \rightarrow \mathbf{F}_p$ . For proofs and more information, see [3, Prop. 11], for example.

### 4. Breaks and levels

(19) Let  $E$  be a cyclic extension of  $K$  of degree  $p$ . The ramification filtration on the group  $G = \text{Gal}(E|K)$  has a unique *break*  $\varepsilon(E)$  (the integer  $j$  such that  $G^j = G$ ,  $G^{j+1} = \{1\}$ ). We have  $\varepsilon(E) = -1$  if and only if  $E$  is unramified over  $K$ ; otherwise,  $\varepsilon(E) > 0$ . We recall what the possibilities for  $\varepsilon(E)$  are, and how it is related to another invariant of  $E$  in some cases.

(20) Suppose that  $e < +\infty$  and  ${}_pK^\times$  has order  $p$ , and let  $D \subset \overline{K}^\times$  be a line (a 1-dimensional subspace). There is a unique integer  $j$  such that

$D \subset \bar{U}_j$  but  $D \not\subset \bar{U}_{j+1}$ , with the convention that  $\bar{U}_0 = \bar{K}^\times$ . We define the *level*  $\delta(D)$  of  $D$  to be  $pc - j$ . We have seen that  $\delta(D) \in [0, pc]$ , and if  $\delta(D) \equiv 0 \pmod{p}$ , then  $\delta(D) = 0$  or  $\delta(D) = pc$ .

(21) Suppose that  $e = +\infty$  and let  $D \subset \bar{K}^+$  be a line. There is a unique integer  $j$  such that  $D \subset \bar{\mathfrak{p}}^j$  but  $D \not\subset \bar{\mathfrak{p}}^{j+1}$ , and we define the *level*  $\delta(D)$  of  $D$  to be  $-j$ . We have  $\delta(D) \in [0, +\infty[$ , and if  $\delta(D) \equiv 0 \pmod{p}$ , then  $\delta(D) = 0$ , as we have seen.

(22) Suppose that  $e < +\infty$  and  ${}_pK^\times$  has order  $p$  (resp.  $e = +\infty$ ). Let  $E$  be a cyclic extension of  $K$  of degree  $p$ , and let  $D \subset \bar{K}^\times$  (resp.  $D \subset \bar{K}^+$ ) be the line such that  $E = K(\sqrt[p]{D})$  (resp.  $E = K(\wp^{-1}(D))$ ). If  $E$  is unramified over  $K$ , then  $D = \bar{U}_{pc}$  (resp.  $D = \bar{\mathfrak{o}}$ ). If  $E$  is ramified over  $K$ , then  $\varepsilon(E) = \delta(D)$ .  $\square$

(23) It follows that if  $E$  is ramified over  $K$ , then  $\varepsilon(E) = b_p^{(i)}$  for some  $i \in [1, e]$  or  $\varepsilon(E) = pc$  (resp.  $\varepsilon(E) = b_p^{(i)}$  for some  $i > 0$ ), and all these possibilities do occur.

(24) *Remark.* The only case not covered by this proposition is when  $e < +\infty$  and  ${}_pK^\times$  is trivial. One can compute  $\varepsilon(E)$  in this case by replacing  $K$  by  $K' = K(\sqrt[p]{1})$  and  $E$  by  $E' = EK'$ . If  $E$  is ramified over  $K$ , then  $\varepsilon(E) = b_p^{(i)}$  for some  $i \in [1, e]$ , and all these possibilities do occur. In particular,  $\varepsilon(E) \not\equiv 0 \pmod{p}$ , as in the case  $e = +\infty$ . See [2, Prop. 63], for example. In all three cases, there are only finitely many  $E$  with a given  $\varepsilon(E)$ , and their number can be easily computed.

## 5. Orthogonality

(25) Recall that for every galoisian extension  $M$  of  $K$ , the profinite group  $G = \text{Gal}(M|K)$  comes equipped with a decreasing filtration  $(G^t)_{t \in [-1, +\infty[}$  — the ramification filtration in the upper numbering — by closed normal subgroups which is separated ( $\cap_t G^t = \{1\}$ ) and exhaustive ( $G^{-1} = G$ ). We put  $G^{t+} = \cup_{s>t} G^s$ , and call  $t$  a ramification break for  $G$  if  $G^{t+} \neq G^t$ , as in the case of degree- $p$  cyclic extensions above. In general,  $G^0$  is the inertia subgroup of  $G$  and  $G^{0+}$  is the (wild) ramification subgroup.

(26) We take  $M$  to be the maximal abelian extension of  $K$  of exponent  $p$  and determine the ramification breaks of  $G$ . When  $e < +\infty$  and  ${}_pK^\times$  has order  $p$  (resp.  $e = +\infty$ ), so that  $M = K(\sqrt[p]{K^\times})$  (resp.  $M = K(\wp^{-1}(K))$ ), we have the pairing

$$G \times \bar{K}^\times \rightarrow {}_pK^\times \text{ (resp. } G \times \bar{K}^+ \rightarrow \mathbf{F}_p),$$

as recalled in the Introduction, and we show that the filtration on  $\bar{K}^\times$  (resp.  $\bar{K}^+$ ) is orthogonal to the filtration on  $G$  in a certain precise sense.

(27) The maximal tamely ramified extension  $M_1$  of  $K$  in  $M$  is the unramified degree- $p$  extension  $M_1 = K(\sqrt[p]{U_{pc}})$  (resp.  $M_1 = K(\wp^{-1}(\mathfrak{o}))$ ), as we have recalled.

(28) Suppose that  $e < +\infty$  and  ${}_pK^\times$  has order  $p$ . We have  $G^t = G^1$  for  $t \in ]-1, 1]$ , and, for  $t \in ]0, pc + 1]$ ,

$$G^{t\perp} = \bar{U}_{pc - \lceil t \rceil + 1}$$

under  $G \times \bar{K}^\times \rightarrow {}_pK^\times$ . The positive ramification breaks in the filtration on  $G$  occur precisely at the  $b_p^{(i)}$  ( $i \in [1, e]$ ) and at  $pc$ .  $\square$

(29) Suppose that  $e = +\infty$ . We have  $G^t = G^1$  for  $t \in ]-1, 1]$ , and, for  $t > 0$ ,

$$G^{t\perp} = \overline{\mathfrak{p}^{-\lceil t \rceil + 1}}$$

under  $G \times \bar{K}^+ \rightarrow \mathbf{F}_p$ . The positive ramification breaks in the filtration on  $G$  occur precisely at the  $b_p^{(i)}$  ( $i > 0$ ).  $\square$

(30) *Remark.* For the proofs, see [2] and [3] respectively. Now suppose that  $e < +\infty$  and  ${}_pK^\times$  is trivial, and put  $K' = K(\sqrt[p]{1})$ ,  $\Gamma = \text{Gal}(K'|K)$  and  $M' = MK'$ . Let  $\omega : \Gamma \rightarrow \mathbf{F}_p^\times$  be the cyclotomic character giving the action of  $\Gamma$  on  ${}_pK'^\times$ . It can be checked that the subspace  $D \subset K'^\times/K'^{\times p}$  such that  $M' = K'(\sqrt[p]{D})$  is precisely the  $\omega$ -eigenspace for the action of  $\Gamma$ . Hence or otherwise, one shows that the positive ramification breaks in the filtration on  $G$  occur precisely at the  $b_p^{(i)}$  ( $i \in [1, e]$ ).

(31) Let  $L$  be an abelian extension of  $K$  of exponent  $p$ . It follows from the foregoing and Herbrand's theorem — the ramification filtration in the upper numbering is compatible with the passage to the quotient — that the ramification breaks of  $\text{Gal}(L|K)$  are integers. This is a special case of the Hasse-Arf theorem, valid for all abelian extensions of local fields. The advantage of the direct proof in this special case is that one can specify which integers occur.

## 6. Norms

(32) Suppose that  $e < +\infty$  and  ${}_pK^\times$  has order  $p$  (resp.  $e = +\infty$ ), and let  $i \in [0, pc + 1]$  (rep.  $i \in \mathbf{N}$ ) be an integer. Let  $L_i = K(\sqrt[p]{U_{pc-i+1}})$  (resp.  $L_i = K(\wp^{-1}(\mathfrak{p}^{-i+1}))$ ). Recall that  $L_0 = K$ , and that  $L_1$  is the unramified degree- $p$  extension of  $K$ . The inductive limit of the  $L_i$  (which is nothing but  $L_{pc+1}$  if  $e < +\infty$  and  ${}_pK^\times$  has order  $p$ ) is equal to the maximal abelian extension  $M$  of  $K$  of exponent  $p$ .

(33) We have  $N_{L_i|K}(L_i^\times) = U_i K^{\times p}$  for every  $i \in [0, pc + 1]$  (rep.  $i \in \mathbf{N}$ ), with the convention that  $U_0 = K^\times$ .  $\square$

(34) All this can presumably be proved by studying, as in [4, Chapter V], what the norm maps  $N_{L_i|K} : L_i^\times \rightarrow K^\times$  do to the filtrations. It follows that  $K^\times/N_{L_i|K}(L_i^\times) = \overline{K}^\times/\bar{U}_i$  for every  $i \in [0, pc+1]$  (rep.  $i \in \mathbf{N}$ ). When  $i = 1$ , the surjective valuation on  $K$  induces an isomorphism  $\bar{v} : K^\times/N_{L_1|K}(L_1^\times) \rightarrow \mathbf{Z}/p\mathbf{Z}$ .

## 7. Reciprocity

(35) Keep the previous notation and continue to suppose that  $e < +\infty$  and  ${}_pK^\times$  has order  $p$  (resp.  $e = +\infty$ ). As  $L_1$  is the unramified degree- $p$  extension of  $K$ , the group  $\text{Gal}(L_1|K)$  has a canonical generator  $\sigma$  — the one which reduces to the  $k$ -automorphism  $x \mapsto x^q$  ( $q = \text{Card } k$ ) of the residue field of  $L_1$ .

(36) *There is a unique isomorphism  $\rho_1 : \overline{K}^\times/\bar{U}_1 \rightarrow \text{Gal}(L_1|K)$  such that  $\rho_1(\bar{\pi}) = \sigma$  for every uniformiser  $\pi$  of  $K$ . The kernel of the resulting map  $K^\times \rightarrow \text{Gal}(L_1|K)$  is  $N_{L_1|K}(L_1^\times)$ .*  $\square$

(37) For  $i \in [1, pc+1]$  (resp.  $i > 0$ ), and for every intermediate extension  $K \subset E \subset L_i$  we have the galoisian projection  $\text{Gal}(L_i|K) \rightarrow \text{Gal}(E|K)$ . In particular, we have the projection  $\text{Gal}(L_i|K) \rightarrow \text{Gal}(L_1|K)$ . Recall also that  $K^\times/N_{L_i|K}(L_i^\times) = \overline{K}^\times/\bar{U}_i$ .

(38) *For  $i \in [1, pc+1]$  (resp.  $i > 0$ ), there is a unique isomorphism  $\rho_i$  making the square*

$$\begin{array}{ccc} \overline{K}^\times/\bar{U}_i & \xrightarrow{\rho_i} & \text{Gal}(L_i|K) \\ \downarrow & & \downarrow \\ \overline{K}^\times/\bar{U}_1 & \xrightarrow{\rho_1} & \text{Gal}(L_1|K) \end{array}$$

*commute, and such that for every intermediate extension  $K \subset E \subset L_i$ , the kernel of the map  $K^\times \rightarrow \text{Gal}(E|K)$  deduced from  $\rho_i$  is  $N_{E|K}(E^\times)$ .*  $\square$

(39) Let  $M = K(\sqrt[p]{K^\times})$  (resp.  $M = K(\wp^{-1}(K))$ ) be the maximal abelian extension of  $K$  of exponent  $p$ . It follows from the foregoing that there is a unique isomorphism  $\rho : \overline{K}^\times \rightarrow \text{Gal}(M|K)$  of profinite groups such that the resulting map  $K^\times \rightarrow \text{Gal}(L_1|K)$  takes every uniformiser of  $K$  to  $\sigma$  and such that for every intermediate extension  $K \subset E \subset M$  of finite degree, the kernel of the resulting map  $K^\times \rightarrow \text{Gal}(E|K)$  is  $N_{E|K}(E^\times)$ . Moreover,  $\rho(\bar{U}_i) = \text{Gal}(M|K)^i$  for every  $i \in [1, pc+1]$  (resp.  $i > 0$ ).

## 8. Orthogonality bis

(40) Suppose that  $e < +\infty$  and  ${}_pK^\times$  has order  $p$ . Combining the kummerian pairing  $G \times \overline{K}^\times \rightarrow {}_pK^\times$  with the reciprocity isomorphism  $\rho : \overline{K}^\times \rightarrow G$ , we get the hilbertian pairing  $\overline{K}^\times \times \overline{K}^\times \rightarrow {}_pK^\times$ .

(41) *For every  $i \in [0, pc + 1]$ , the orthogonal complement of  $\bar{U}_i$  for the hilbertian pairing is  $\bar{U}_{pc-i+1}$ .*  $\square$

(42) Suppose finally that  $e = +\infty$ . As before, combining the pairing  $G \times \overline{K}^+ \rightarrow \mathbf{F}_p$  with the reciprocity isomorphism  $\rho : \overline{K}^\times \rightarrow G$ , we get the hilbertian pairing  $\overline{K}^\times \times \overline{K}^+ \rightarrow \mathbf{F}_p$ .

(43) *For every  $i \in \mathbf{N}$ , the orthogonal complement of  $\bar{U}_i$  for the hilbertian pairing is  $\overline{\mathfrak{p}}^{-i+1}$  and vice versa.*  $\square$

(44) *Remark.* In both cases,  $\bar{U}_0 = \overline{K}^\times$  by convention. This Note was written in response to a question by Hatice Boylan on MathOverflow [1].

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